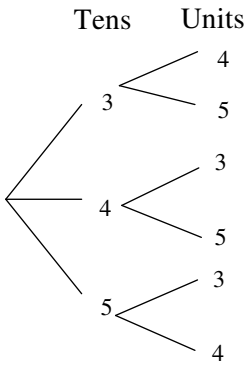
ACE Examination 2011

HSC General Mathematics Yearly Examination

Worked solutions and marking guidelines

Section 1		
	Solution	**Criteria**
1	$3a^2(a+2)-a^2 = 3a^3+6a^2-a^2$ $= 3a^3+5a^2$	1 Mark: B
2	Tax payable = \$81 500 – \$2 500 = \$79 000 Taxable income between \$37 001 and \$80 000 (third line)	1 Mark: C
3	$\text{Revolutions} = \frac{6.6\times1000\times100}{220}$ $= 3000$	1 Mark: D
4	Boys median is 25. Girls median is 26. Difference in medians is 1.	1 Mark: B
5	$\text{Angle size} = \frac{144}{720}\times360 = 72^\circ$	1 Mark: C
6	Exponential function. Check (0, 1) on each graph.	1 Mark: B
7	Categorical data - Mode	1 Mark: C
8	$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$	1 Mark: D
9	$465 = 20\times x + 8\times1.5\times x$ (x is the hourly rate of pay) $= 32x$ $x = \frac{465}{32}$ $= \$14.53$	1 Mark: C
10	$V = \frac{4}{3}\pi r^3$ $360 = \frac{4}{3}\pi r^3$ $r = \sqrt[3]{360\times\frac{3}{4\pi}}$ $= 4.4$	1 Mark: B
11	$A = 800(1.1^t)$ $= 800(1.1^0)$ $= 800$	1 Mark: A
12	Correlation coefficient of about +0.3 is a positive linear relationship with low strength.	1 Mark: B

13	$z=\frac{x-\bar{x}}{s}$ and $z=\frac{x-\bar{x}}{s}$ $=\frac{58-70}{6}$ $=\frac{82-70}{6}$ $=-2$ $=2$ 95% of scores have a z-score between -2 and 2	1 Mark: C
14	Number of outcomes $=7\times6$ $=42$	1 Mark: C
15	$\frac{AC}{\sin 80^\circ}=\frac{15}{\sin 40^\circ}$ $AC=\frac{15\sin 80^\circ}{\sin 40^\circ}$	1 Mark: A
16	Gradient intercept formula: $y=mx+b$. Gradient is 2	1 Mark: C
17	N49ºE	1 Mark: C
18	Events that are very unlikely have a probability close to 0. Best answer is 1%.	1 Mark: B
19	$\cos\theta=\frac{34^2+27.4^2-24.3^2}{2\times34\times27.4}$ $\theta=45^\circ$	1 Mark: B
20	Percentage $=\frac{10}{50}$ $=20\%$	1 Mark: B
21	Total paid $=2200+\$820\times12\times4$ $=\$41560$	1 Mark: D
22	$4\sqrt{m}-9=0$ $\sqrt{m}=\frac{9}{4}$ $m=\frac{9^2}{4}$ $=\frac{81}{16}$	1 Mark: B

Section II		
	Solution	**Criteria**
23(a) (i)	 Sample space is {34, 35, 43, 45, 53, 54}	2 Marks: Correct answer. 1 Mark: One stage of the tree diagram is correct or lists the sample space.
23(a) (ii)	$\text{P}(44) = 0$ (The same number cannot be used twice)	1 Mark: Correct answer.
23(a) (iii)	$\text{P}(45) = \frac{n(45)}{n(s)} = \frac{1}{6}$	1 Mark: Correct answer.
23(a) (iv)	$\text{P}(>50) = \frac{n(53 \text{ or } 54)}{n(s)}$ $= \frac{2}{6} = \frac{1}{3}$	1 Mark: Correct answer.
23(b) (i)	$S = V_0 - Dn$ $235 = 470 - D \times 2$ $D = \frac{470 - 235}{2}$ $= \$117.50$	1 Mark: Correct answer.
23(b) (ii)	$\text{Depreciation rate} = \frac{D}{V_0}$ $= \frac{117.50}{470}$ $= 25\%$	1 Mark: Correct answer.
23(b) (iii)	$\text{Gross annual salary} = 1250 \times 26$ $= \$32\ 500$	1 Mark: Correct answer.
23(b) (iv)	$\text{Taxable income} = 32500 - 117.50$ $= \$32\ 382.50$	1 Mark: Correct answer.

23(b) (v)	Medicare levy $= 1.5\%$ of \$32 382.50 $= 0.015 \times 32382.50$ $= \$485.74$	1 Mark: Correct answer.
23(c) (i)	To find the cross-sectional area of the creek $A = \frac{h}{3}(d_f + 4d_m + d_l) + \frac{h}{3}(d_f + 4d_m + d_l)$ $= \frac{2.7}{3}(0 + 4 \times 3.5 + 2.3) + \frac{2.7}{3}(2.3 + 4 \times 1.3 + 0)$ $= 21.42 \text{ m}^2$ $= 21.4 \text{ m}^2$ Area of the cross-section is 21.4 m^2	2 Marks: Correct answer. 1 mark: Makes some progress using Simpson's rule.
23(c) (ii)	$V = Ah$ $= 21.4 \times 50$ $= 1070 \text{ m}^3$ Volume of water in the creek is 1070 m^3	1 Mark: Correct answer.
24(a) (i)	25 students were surveyed	1 Mark: Correct answer.
24(a) (ii)	Number in 44.5 class $= 20 - 15$ $= 5$ There were 5 students in the 44.5 class.	1 Mark: Correct answer.
24(a) (iii)	The median is about 35 (from the graph)	1 Mark: Correct answer.
24(b) (i)	$A = \pi r^2$ $= \pi \times 8^2$ $= 201.0619298$ $= 201 \text{ m}^2$ Area of the garden is 201 square metres	1 Mark: Correct answer.
24(b) (ii)	$A = \pi(R^2 - r^2)$ $= \pi \times (14^2 - 8^2)$ $= 414.6902303$ $= 415 \text{ m}^2$ Area to be resurfaced is 415 square metres	1 Mark: Correct answer.

24(c)	Financial expectation = Sum[P(E)×Financial outcome] $\$0 = (\frac{1}{100}\times 250)+(\frac{5}{100}\times 25)+(\frac{1}{100}\times x)+(\frac{100}{100}\times -5)$ $= 2.5+1.25+0.01x-5$ $0.01x = 1.25$ $x = \$125$ Mystery prize is \$125 Alternatively $(100\times 5)-(250\times 1)-(25\times 5) = \$125$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.
24(d) (i)	\$50 000	1 Mark: Correct answer.
24(d) (ii)	81 months (allow 79 to 83)	1 Mark: Correct answer.
24(d) (iii)	64 months (allow 62 to 66)	1 Mark: Correct answer.
24(d) (iv)	Interest = $64\times 1000-50000$ = \$14 000	1 Mark: Correct answer.
24(e)	$3(a-2) = 5-a$ $3a-6 = 5-a$ $4a = 11$ $a = \frac{11}{4}$ $= 2.75$	2 Marks: Correct answer. 1 Mark: Correctly expands grouping symbols or makes similar progress
25(a) (i)	$S = V_0(1-r)^n$ $= 120000\times(1-0.16)^3$ = \$71 124.48 Value of the delivery truck is \$71 124.85.	2 Marks: Correct answer. 1 Mark: Substitutes a correct value into declining balance formula

25(a) (ii)	$\text{Loss} = 120000 - 71124.48$ $= \$48\ 875.52$ $\text{Percentage Loss} = \frac{48875.52}{120000} \times 100$ $= 40.7296\%$ $= 41\%$	1 Mark: Correct answer.
25(b) (i)	$\text{Number of arrangements} = 26 \times 26 \times 26 \times 10 \times 10 \times 10$ $= 17\ 576\ 000$	1 Mark: Correct answer.
25(b) (ii)	$\text{P(BON007)} = \frac{1}{17\ 576\ 000}$	1 Mark: Correct answer.
25(b) (iii)	$\text{P(007)} = \frac{1}{17\ 576}$	1 Mark: Correct answer.
25(c) (i)	$50 + 60 + 25 + 20 = 155$	1 Mark: Correct answer.
25(c) (ii)	$\text{Percentage} = \frac{60}{80}$ $= 75\%$	1 Mark: Correct answer.
25(c) (iii)	$\text{Fraction} = \frac{25}{75}$ $= \frac{1}{3}$	1 Mark: Correct answer.
25(c) (iv)	$\text{Probability} = \frac{20}{155}$ $= \frac{4}{31}$	1 Mark: Correct answer.
25(d) (i)	Intersection value is 3.7908 (10% and 5 years) $PV = 3.7908 \times 11000$ $= \$41\ 698.80$	1 Mark: Correct answer.
25(d) (ii)	Intersection value is 3.9020 (1% and 4 years) $PV = 3.9020 \times 8000$ $= \$31\ 216$	1 Mark: Correct answer.

25(d) (iii)	Intersection value is 2.5771 (8% and 3 years) $47934 = x \times 2.5771$ $x = \dfrac{47934}{2.5771}$ $= \$18\ 600$ Value of the annuity is \$18 600 per year.	1 Mark: Correct answer.
26(a) (i)	$d = 5\sqrt{\dfrac{h}{2}}$ $= 5\sqrt{\dfrac{18.5}{2}}$ $= 15.2$ km	1 Mark: Correct answer.
26(a) (ii)	$d = 5\sqrt{\dfrac{h}{2}}$ $\dfrac{d}{5} = \sqrt{\dfrac{h}{2}}$ $\dfrac{h}{2} = \dfrac{d^2}{25}$ $h = \dfrac{2d^2}{25}$	1 Mark: Correct answer.
26(a) (iii)	$h = \dfrac{2d^2}{25}$ $= \dfrac{2 \times 8^2}{25}$ $= 5.12$ m	1 Mark: Correct answer.
26(b) (i)	$\angle AOB = 133 - 50$ $= 83^\circ$	1 Mark: Correct answer.
26(b) (ii)	$\angle AOC = (360 - 305) + 50$ $= 105^\circ$	1 Mark: Correct answer.
26(b) (iii)	$AB^2 = 40^2 + 55^2 - 2 \times 40 \times 55 \times \cos 83^\circ$ $AB^2 = 4088.774889$ $AB = 63.94$ m Distance from A to B is 4.25 m	2 Marks: Correct answer. 1 Mark: Recognises the use of the cosine rule and makes progress.

26(b) (iv)	$A = \frac{1}{2}ab\sin\angle AOB$ $= \frac{1}{2}\times 40\times 55\times \sin 83^\circ$ $= 1091.800767$ $= 1092\ \text{m}^2$	1 Mark: Correct answer.
26(c) (i)	Longitude difference $= 153-18$ $= 135^\circ$ Time difference $= 135\times 4$ $= 540$ minutes $= 9$ h	1 Mark: Correct answer.
26(c) (ii)	Springbok 18° E — Coffs Harbour 153° E □ West — East + Time Springbok = 3 p.m. □ 9 h = 6 a.m	1 Mark: Correct answer.
26(d) (i)	$V = \pi r^2 h$ $= \pi\times 2.5^2\times 3$ $= 58.90486225$ $= 58.9\ \text{m}^3$	1 Mark: Correct answer.
26(d) (ii)	$V = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\times\pi\times 2.5^2\times 3$ $= 19.63495408$ $= 19.6\ \text{m}^3$	1 Mark: Correct answer.
26(d) (iii)	$V = 58.904... + 19.643...$ $= 78.53981633$ $= 78.5\ \text{m}^3$	1 Mark: Correct answer.
27(a) (i)	$\text{P(W)} = \frac{2}{5}$	1 Mark: Correct answer.
27(a) (ii)	$\text{P(LW)} = \frac{3}{5}\times\frac{2}{5}$ $= \frac{6}{25}$	1 Mark: Correct answer.

27(a) (iii)	$\text{P(LL)} = \frac{3}{5} \times \frac{3}{5}$ $= \frac{9}{25}$	1 Mark: Correct answer.
27(a) (iv)	$\text{P(E)} = 1 - \text{P(LL)}$ $= 1 - \frac{9}{25}$ $= \frac{16}{25}$ or $\text{P(E)} = \text{P(WW)} + \text{P(LW)} + \text{P(WL)}$ $= \frac{4}{25} + \frac{6}{25} + \frac{6}{25}$ $= \frac{16}{25}$	1 Mark: Correct answer.
27(b) (i)	$N = \frac{k}{A}$ $4800 = \frac{k}{200}$ $k = 960\,000$ $N = \frac{960\,000}{A}$ $= \frac{960\,000}{240}$ $= 4\,000$ The venue could hold 4000 people.	2 Marks: Correct answer. 1 Mark: Calculates the proportionality constant.
27(b) (ii)	$N = \frac{960\,000}{A}$ $5000 = \frac{960\,000}{A}$ $A = 192$ The space allowed is 192 cm^2	1 Mark: Correct answer.
27(c) (i)	$FV = M\left[\frac{(1+r)^n - 1}{r}\right]$ $= 1200 \times \left[\frac{(1+\frac{0.09}{12})^{60} - 1}{\frac{0.09}{12}}\right]$ $= \$90\,508.96$	2 Marks: Correct answer. 1 Mark: Recognises the future value formula and uses 1200 for M.
27(c) (ii)	$\text{Interest} = 90508.96 - (1200 \times 60)$ $= \$18\,508.96$	1 Mark: Correct answer.
27(d) (i)	AC makes an angle of 308º with the north direction. $\angle ACB = 308^\circ - 270^\circ$ $= 38^\circ$	1 Mark: Correct answer.

27(d) (ii)	To find $\angle BAC$ $\angle BAC + 105^\circ + 38^\circ = 180^\circ$ $\angle BAC = 37^\circ$ $\frac{a}{\sin A} = \frac{b}{\sin B}$ $\frac{AC}{\sin 105^\circ} = \frac{60}{\sin 37^\circ}$ $AC = \frac{60 \times \sin 105^\circ}{\sin 37^\circ}$ $= 96.30126758$ $= 96.3$ m	2 Marks: Correct answer. 1 Mark: Correctly calculates $\angle BAC$ or makes some progress in the use of the sine rule.
28(a) (i)	Company X: Range = 232, IQR = 87 Company Y: Range = 59, IQR = 22.5	1 Mark: Correct answer.
28(a) (ii)	Company X: Mean = 87.25, Median = 60 Company Y: Mean = 77.125, Median = 73.5	1 Mark: Correct answer.
28(a) (iii)	Median is a better measure. Mean in Company X has been distorted by an outlier (252).	1 Mark: Correct answer.
28(a) (iv)	Staff numbers of company Y is more than the staff numbers of company X. There is a greater spread in the staff numbers of company X compared to the staff numbers of company Y.	1 Mark: Correct answer.
28(b) (i)	Shortest distance is along the meridian of longitude. Latitude difference = $16 - 1$ $= 15^\circ$ One degree on a great circle is 60 nautical miles Distance = 15×60 = 900 M	2 Marks: Correct answer. 1 Mark: Correctly calculates the latitude difference.

28(b) (ii)	$1 \text{ Knot} = \dfrac{1 \text{ Nautical mile}}{1 \text{ Hour}}$ $9 = \dfrac{900}{t}$ $t = \dfrac{900}{9}$ $= 100 \text{ hours}$	1 Mark: Correct answer.
28(b) (iii)	Vanuatu and Nauru are located on the same meridian of longitude. No time difference between these Pacific islands. Travel time = 100 h $= 4$ days and 4 h Yacht arrives at 11.00 am on Friday 16 October.	1 Mark: Correct answer.
28(c) (i)	Correlation coefficient is about +0.3	1 Mark: Correct answer.
28(c) (ii)	Median of Group 1 is (10, 20) Median of Group 2 is (27.5, 25) Median of Group 3 is (45, 45)	1 Mark: Correct answer.
28(c) (iii)	Equation is an approximation based on the graph $y = mx + b$ $p = \dfrac{3}{4}s + 10$ Accept any equations close to this value.	1 Mark: Correct answer.
28(c) (iv)	About 37 (accept 36 to 38)	1 Mark: Correct answer.